Approximate Solution of a Non-Linear System of Equations with Maple Mathematical Packages

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Abstract

In this article, a sequence of analytical and approximate solutions of a system of Nonlinear equations using the Maple mathematical package is presented, and shows the application of a system of Nonlinear equations, for example, in physical and mechanical processes the problem of numerical solution of problems is considered. It is known that a number of approximate calculation methods for solving systems of nonlinear equations include Newton's method and iteration methods. In this article, a number of specific practical problems have been solved using these methods, an algorithm and a block diagram have been developed, and a program has been developed in the Maple mathematical package. The obtained results were compared with analytical solutions, graphs were used in the form of results, and appropriate conclusions were drawn.

Keywords: Maple mathematical package, system of Nonlinear equations, Newton's method, iteration method, mathematical model

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Introduction

One of the areas of application of computers remains the study of mathematical, mechanical and physical processes and mathematical models of objects using computational methods and computer software. The methods of computational mathematics and the modern capabilities of computers together serve to reveal the hitherto unknown features of such processes and objects, and at the same time to improve technological processes. These processes also depend on the application of computational mathematics and computers in scientific research and are of scientific and practical importance.

Today, with the development of science and technology, the role of mathematics and computers is increasing. Mathematics is used in physics, mechanics, biology, chemistry and astronomy, as well as in solving economic problems, analyzing these processes, and many other areas. Part of the mathematical model of processes in these fields is reduced to a system of nonlinear equations. The role of mathematical packages in solving such nonlinear problems is invaluable. In particular, the problem of analytical and approximate solution of a system of nonlinear equations using the Maple mathematical package simplifies the solution of such problems.

Main part

We see this in the example of solving this problem below. When solving this problem, the formulation of the problem and a sequential algorithm for its solution are presented, as well as the necessary computational methods for solving a system of nonlinear equations are described. In order to solve the problem, first of all, the system of nonlinear equations, their solution by analytical methods, in which cases mathematical packages can be used, and the approximate solution of nonlinear equations of a number of models of physical and mechanical processes are considered and problems are discussed.

Problem:

Calculate the solution of the following nonlinear equations in the Newton's method with an accuracy of $\varepsilon = 0.001$.

$$\begin{cases} f_1(x, y) = 2x^3 - y^2 - 1 = 0 \\ f_2(x, y) = xy^3 - y - 4 = 0 \end{cases}$$

Solution.

The fact that the system of equations given in this example has a single positive real solution can be seen from the graphs of the functions $f_1(x, y) = 0$ and $f_2(x, y) = 0$ using the implication function of the plots Maple program package below (Figure 1):
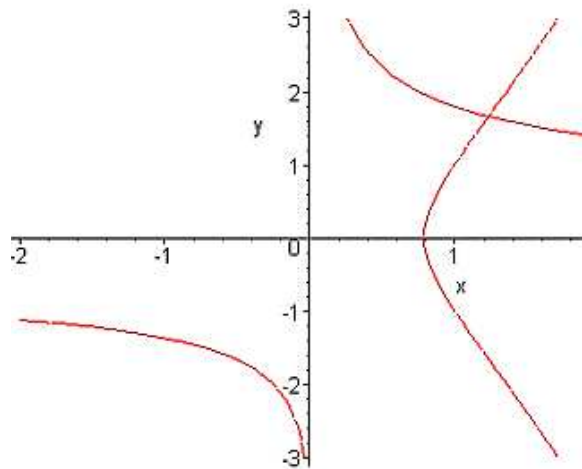
Figure 1.*Determining the initial approximation of the root of a system of nonlinear equations given in the example graphically using the Maple program.*

$$> plots\,[implicitplot]\,](\{2^\wedge * x^\wedge \wedge 3 - y^\wedge \wedge 2 - 1 = 0, x^\wedge * y^\wedge \wedge 3 - y - 4 = 0\}, x = -2..2, y = -3..3| \,);$$

According to this method, let the initial approximation be

$x_0 = 1{,}2; \quad y_0 = 1{,}7.$

In that case:

$$\Delta(x_0, y_0) = \begin{vmatrix} 6x_0^2 & -2y_0 \\ y_0^3 & 3x_0 y_0^2 - 1 \end{vmatrix} \text{ or } \Delta(1{,}2; 1{,}7) = \begin{vmatrix} 8{,}64 & -3{,}40 \\ 4{,}91 & 9{,}40 \end{vmatrix} = 97{,}910. \text{ According }$$

to

formula:

$$\left. \begin{array}{l} x_1 = 1{,}2 - \dfrac{1}{97{,}91}\begin{vmatrix} 0{,}434 & -3{,}40 \\ 0{,}1956 & 9{,}40 \end{vmatrix} = 1{,}2 + 0{,}0349 = 1{,}2349 \\[2mm] y_1 = 1{,}7 - \dfrac{1}{97{,}91}\begin{vmatrix} 8{,}64 & -0{,}434 \\ 4{,}91 & 0{,}1956 \end{vmatrix} = 1{,}7 - 0{,}0390 = 1{,}6610 \end{array} \right\}$$

Continuing the calculations in the same way, we find $x_2 = 1{,}2343 \quad y_2 = 1{,}6615$ and continue the calculations with the required accuracy. Let's find the only real solution of this system of equations analytically using the Maple program:

>solve ($\{2^* x^\wedge 3 - y^\wedge 2 - 1 = 0, x^* y^\wedge 3 - y - 4 = 0\}, \{x, y\}$);   allvalues (%);   evalf (%);

$$\{x = 1.234274484y = 1.661526467\}$$

It is seen from the results that approximate solution $x_2 = 1{,}2343 y_2 = 1{,}6615$ be considered found with sufficient accuracy ε .

Now let's look at a numerical solution to this problem in the Maple system. First, we compute the Jacob matrix using the **jacobian** function of the **linalg** package, and then compute its **inverse** function using the inverse function of the linalg package. The **eval** function returns the numeric value of an expression. The **evalm** function performs operations on matrices and vectors to produce numeric results. Since the initial vectors **xx** and **eps** are degrees of precision, we

perform approximate calculations using Newton's method:

$>$ with(linalg):

$F:=(x,y)->[2*x\char94 3-y\char94 2-1, x*y\char94 3-y-4];$

FP: =jacobian $(F(x,y),[x,y])$; FPINV:=inverse(FP);

xx:=[1.2,1.7]; eps:=0.0001; Err:=1000; v:=xx; v1:=[1e10,1e10];

j:=0:

for i while Err $>$ eps do

v1:=eval(v);

$M:= eval(eval(FPINV),[x = v[1], y = v[2]])$

$\mathbf{v}:= evalm\,(\mathbf{v} - M\& * \mathbf{F}(\mathbf{v}[1], v[2]));$

Err:=max(abs(v1[1]-v[1]),abs(v1[2]-v[2]));

j:=j+1;

end do;

The result of the calculation is as follows:

$$F:=(x,y) \to [2x^3 - y^2 - 1, xy^3 - y - 4]$$

$$FP := \begin{bmatrix} 6x^2 & -2y \\ y^3 & 3xy^2 - 1 \end{bmatrix}$$

$$FPIN := \begin{bmatrix} \dfrac{3xy^2 - 1}{2(9x^3y^2 - 3x^2 + y^4)} & \dfrac{y}{9x^3y^2 - 3x^2 + y^4} \\ -\dfrac{y^3}{2(9x^3y^2 - 3x^2 + y^4)} & \dfrac{3x^2}{9x^3y^2 - 3x^2 + y^4} \end{bmatrix}$$

$$xx := [1.2, 1.7]$$
$$eps := 0.0001$$
$$Er := 1000$$
$$V := [1.2, 1.7]$$
$$v1 := [0.110^{11}, 0.110^{11}]$$
$$v1 := [1.2, 1.7]$$
$$M := \begin{bmatrix} 0.09600350200 & 0.03470990077 \\ -0.05015580660 & 0.08820398313 \end{bmatrix}$$
$$v := [1.234876263, 1.660979681]$$
$$Er := 0.039020319$$
$$j := 1$$
$$V1 := [1.234876263, 1.660979681]$$
$$V := \begin{bmatrix} 0.09258867450 & 0.03335772210 \\ -0.04601453420 & 0.09187560387 \end{bmatrix}$$
$$v := [1.234274675, 1.661526276]$$
$$En := 0.000601588$$
$$j := 2$$
$$V := \begin{bmatrix} 0.09264916080 & 0.03338417877 \\ -0.04608134315 & 0.09182868696 \end{bmatrix}$$
$$v := [1.234274484, 1.661526467]$$
$$EJ := 0.19110^{-6}$$
$$J := 3$$

The result shows that the solution given in step 3 of the calculation process has been achieved. The improved versions of the Newton's method are very effective, but the speed of convergence of these methods increases sharply when this initial approximation is taken very close to the isolated root and the conditions of approximation are satisfied.

Thus, the choice of the correct approximate method and initial condition depending on the type of practical problem for which the problem of solving a system of nonlinear equations is being solved, the effective use of these methods and mathematical packages increases the speed of approaching a certain solution.

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