ACADEMIC JOURNAL

Academic Journal of Digital Economics and Stability
Volume 9, 2021

ISSN 2697-2212 Online: https://academicjournal.io

Cauchy Problem for Second-Order Linear Non-Homogeneous Differential Equations with Impulse Effect

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Abstract: The second-order, variable-coefficient linearly homogeneous pulse effect is not homogeneous

$$\frac{d^2x}{dt^2}+p(t)\frac{dx}{dt}+q(t)x=f(t),\quad t\notin D,$$

$$\Delta_1\big(x(t+0),x(t-0)\big)=h(t),\ \ t\in D,$$

$$\Delta_2\big(x'(t+0),x'(t-0)\big)=g(t),\ \ t\in D$$

for the differential equation in the form

$$x(0)=x_0,\quad x'(0)=x'_0$$

the problem of finding a solution that satisfies the initial condition is studied.

Keywords: Impulse differential equation, Cauchy problem, system of fundamental solutions

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Well, we don't have a second-order, generally linear homogeneous coefficient with a pulse effect.

$$\frac{d^2x}{dt^2}+p(t)\frac{dx}{dt}+q(t)x=f(t),\quad t\notin D, \tag{1}$$

$$\Delta_1\left(x(t+0),x(t-0)\right)=h(t),\ \ t\in D, \tag{2}$$

$$\Delta_2\left(x'(t+0),x'(t-0)\right)=g(t),\ \ t\in D$$

for the differential equation in the form

$$x(0)=x_0,\quad x'(0)=x_0' \tag{3}$$

Let us consider the problem of finding a solution that satisfies the initial condition, where $p(t)$, $q(t)$ and $f(t)$ for the functions $0\le t$ defined and continuous functions, if the set has D an impulse effect (1) - (3) with a graph of the $0\le t$ solution of the Cauchy problem the points $x=\psi(t)$ of intersection of the graph of a (t,x) function, i.e. the set of $\left\{(t,x)\in R^2,\ x(t)=\psi(t)\right\}$ points,

Wells Δ_1 and Δ_2 functions are such functions,

$$\begin{cases}\Delta_1(x,y)=0,\\ \Delta_2(x,y)=0\end{cases} \tag{4}$$

Let the system be solved with one value. This condition (1) - (3) ensures that the Cauchy problem is unique.

We know that if $x^{(1)}(t)$ and $x^{(2)}(t)$ are a system of fundamental solutions to equation (1), then the general solution of equation (1)

$$x(t)=C_1x^{(1)}(t)+C_2x^{(2)}(t)+x_{\ddot{a}\grave{a}\eth\grave{a}}(t) \tag{5}$$

(1) - (3) is the solution of the Cauchy problem with the impulse effect we are looking at, C_1 and C_2 we have the same view that the appropriate values are selected.

Let (1) be the solution of equation (3) that satisfies the initial conditions

$$x_0(t)=C_1^{(0)}x^{(1)}(t)+C_2^{(0)}x^{(2)}(t)+x_{\ddot{a}\grave{a}\eth\grave{a}}(t) \tag{6}$$

let it be Then on the condition of the issue

$$C_1^{(0)}x^{(1)}(t)+C_2^{(0)}x^{(2)}(t)+x_{\ddot{a}\grave{a}\eth\grave{a}}(t)=\psi(t)$$

At the smallest value that satisfies the condition of satisfying the equation (1), equation (1) is

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first affected by the impulse. Let's say this is the value of

$$C_1^{(0)}x^{(1)}(t_1)+C_2^{(0)}x^{(2)}(t_1)+x_{\ddot{a}\grave{a}\check{\delta}\grave{a}}(t_1)=\psi(t_1)\ .$$

The solution after the effect of the impulse is sought again in the form (5), but the arbitrary variables and the values of

$$\begin{cases}\Delta_1(x(t_1),x_0(t_1))=0,\\ \Delta_2(x'(t_1),x_0'(t_1))=0\end{cases}$$

determined from the system. (4) because of the condition

$$\begin{cases}\Delta_1(C_1x^{(1)}(t_1)+C_2x^{(2)}(t_1)+x_{\ddot{a}\grave{a}\check{\delta}\grave{a}}(t_1),x_0(t_1))=0,\\ \Delta_2(C_1x^{(1)}{}'(t_1)+C_2x^{(2)}{}'(t_1)+x'_{\ddot{a}\grave{a}\check{\delta}\grave{a}}(t_1),x_0'(t_1))=0\end{cases}$$

is solved with one value in relation to the system $C_1x^{(1)}(t_1)+C_2x^{(2)}(t_1)+x_{\ddot{a}\grave{a}\check{\delta}\grave{a}}(t_1)$ and $C_1x^{(1)}{}'(t_1)+C_2x^{(2)}{}'(t_1)+x'_{\ddot{a}\grave{a}\check{\delta}\grave{a}}(t_1)$ expressions, i.e.

$$\begin{cases}C_1x^{(1)}(t_1)+C_2x^{(2)}(t_1)+x_{\ddot{a}\grave{a}\check{\delta}\grave{a}}(t_1)=h_1,\\ C_1x^{(1)}{}'(t_1)+C_2x^{(2)}{}'(t_1)+x'_{\ddot{a}\grave{a}\check{\delta}\grave{a}}(t_1)=g_1\end{cases}$$

or

$$\begin{cases}C_1x^{(1)}(t_1)+C_2x^{(2)}(t_1)=h_1-x_{\ddot{a}\grave{a}\check{\delta}\grave{a}}(t_1),\\ C_1x^{(1)}{}'(t_1)+C_2x^{(2)}{}'(t_1)=g_1-x'_{\ddot{a}\grave{a}\check{\delta}\grave{a}}(t_1).\end{cases}$$

In this case, since $x^{(1)}(t)$ and $x^{(2)}(t)$ is a system of fundamental solutions to the homogeneous equation in equation (1), we have the same value for the last system C_1 and C_2 the solution, $C_1=C_1^{(1)}$ and $C_2=C_2^{(1)}$ we have these unknown coefficients and values. As a result, (1) - (3) has a pulse effect after the first pulse effect of the Cauchy problem

$$x_1(t)=C_1^{(1)}x^{(1)}(t)+C_2^{(1)}x^{(2)}(t)+x_{\ddot{a}\grave{a}\check{\delta}\grave{a}}(t)$$

we have a solution.

This solution lasts until the second impulse. Second impulse effect

$$C_1^{(1)}x^{(1)}(t)+C_2^{(1)}x^{(2)}(t)+x_{\ddot{a}\grave{a}\check{\delta}\grave{a}}(t)=\psi(t)$$

takes place at the smallest value that satisfies the condition of satisfying the equality. Let's say this is the value of

$$C_1^{(1)}x^{(1)}(t_2)+C_2^{(1)}x^{(2)}(t_2)+x_{\ddot{a}\grave{a}\check{\delta}\grave{a}}(t_2)=\psi(t_2)\ .$$

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The solution after the second impulse is sought again in view (5), but the arbitrary variables and the values of are now

$$\begin{cases} \Delta_1(x(t_2), x_1(t_2)) = 0, \\ \Delta_2(x'(t_2), x_1'(t_2)) = 0 \end{cases}$$

determined from the system. Using conditions (2) and (4)

$$\begin{cases} \Delta_1(C_1 x^{(1)}(t_2) + C_2 x^{(2)}(t_2) + x_{äàðà}(t_2), x_1(t_2)) = 0, \\ \Delta_2(C_1 x^{(1)}{}'(t_2) + C_2 x^{(2)}{}'(t_2) + x'_{äàðà}(t_2), x_1'(t_2)) = 0 \end{cases}$$

the system $C_1 x^{(1)}(t_2) + C_2 x^{(2)}(t_2) + x_{äàðà}(t_2)$ and $C_1 x^{(1)}{}'(t_2) + C_2 x^{(2)}{}'(t_2) + x'_{äàðà}(t_2)$ its expressions are solved again with the same value, i.e.

$$\begin{cases} C_1 x^{(1)}(t_2) + C_2 x^{(2)}(t_2) + x_{äàðà}(t_2) = h_2, \\ C_1 x^{(1)}{}'(t_2) + C_2 x^{(2)}{}'(t_2) + x'_{äàðà}(t_2) = g_2 \end{cases}$$

Or

$$\begin{cases} C_1 x^{(1)}(t_2) + C_2 x^{(2)}(t_2) = h_2 - x_{äàðà}(t_2), \\ C_1 x^{(1)}{}'(t_2) + C_2 x^{(2)}{}'(t_2) = g_2 - x'_{äàðà}(t_2). \end{cases}$$

Solving another value with respect to this system and lar, we obtain these unknown coefficients and values. As a result, (1) - (3) has a pulse effect after the second pulse effect of the Cauchy problem

$$x_2(t) = C_1^{(2)} x^{(1)}(t) + C_2^{(2)} x^{(2)}(t) + x_{äàðà}(t)$$

we have a solution.

If we continue the process in this way, then the solution after the effect of the impulse

$$x_n(t) = C_1^{(n)} x^{(1)}(t) + C_2^{(n)} x^{(2)}(t) + x_{äàðà}(t),\ (n > 2)$$

It would be appropriate to have, where and the coefficients

$$\begin{cases} C_1 x^{(1)}(t_n) + C_2 x^{(2)}(t_n) = h_n - x_{äàðà}(t_n), \\ C_1 x^{(1)}{}'(t_n) + C_2 x^{(2)}{}'(t_n) = g_n - x'_{äàðà}(t_n) \end{cases}$$

is determined by solving the system, and if

$$\begin{cases} \Delta_1(C_1 x^{(1)}(t_n) + C_2 x^{(2)}(t_n) + x_{äàðà}(t_n),\ x_{n-1}(t_n)) = 0, \\ \Delta_2(C_1 x^{(1)}{}'(t_{n2}) + C_2 x^{(2)}{}'(t_n) + x'_{äàðà}(t_n),\ x'_{n-1}(t_n)) = 0 \end{cases}$$

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The values of the corresponding $C_1 x^{(1)}(t_n) + C_2 x^{(2)}(t_n) + x_{äàðà}(t_n)$ and $C_1 x^{(1)}{}'(t_n) + C_2 x^{(2)}{}'(t_n) + x'_{äàðà}(t_n)$ expressions that satisfy the system, as well as the value of

$$C_1^{(n-1)} x^{(1)}(t_n) + C_2^{(n-1)} x^{(2)}(t_n) + x_{äàðà}(t_n) = \psi(t_n)$$

is determined from the condition of satisfying the equality.

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