

Theory and System Selection of Data Taxation

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Abstract

In the context of the digital economy, it is an inevitable direction for tax reform to include data as a key factor of production in tax collection. At present, the corresponding data tax does not exist. This paper analyzes the taxability of data based on the connotation characteristics and value creation of digital assets. Using the game theory model, it discusses the tax system design of digital data and compares the impact on the market of the data tax that calculates the tax base based on the quantity or value of data. This research will help guide the establishment of a data taxation system and make different institutional choices in the formulation of taxation on data.

Keywords: Data Taxation.

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1. Introduction

In the midst of the epidemic, the digital economy represented by big data applications will usher in major development opportunities. During the epidemic, the digital economy, such as telecommuting, online education, and live streaming, has developed rapidly. The impact of this new contactless service model on economic development and government governance is long-term and far-reaching (Brem, Viardot et al. 2021).

Data are the basic and strategic resource of the digital economy. Today, big data collection, processing, analysis, mining and other technologies have been widely used in business, finance, manufacturing, research and development, government services, public security and other fields. The concept of big data has penetrated into every corner of social practice and even become an important factor for enterprises to form a dominant market position.

However, at present, the theoretical understanding of big data still lags behind its application practice (Sivarajah, Kamal et al. 2017). Moreover, there are also many disputes over big data ownership, transaction pricing and capitalization, which pose potential hidden dangers for the sustainable and healthy development of the big data industry and digital economy and restrict the transformation of big data into production factors (Tene and Polonetsky 2012). With the blessing of artificial intelligence, the Internet of Things, cloud computing, fifth-generation mobile communications and other technologies, the development of big data applications and the big data industry has entered an unstoppable period. However, history has proven that a market with explosive growth is not necessarily a market with sustainable development. With the rapid expansion of big data and related industries, some basic theoretical issues need to be re-examined. The capitalization of big data and the pricing of data products are inseparable from business operations. How should the government use taxation methods to turn this spillover into a public product that feeds back the "source" of big data, that is, the individual generating information? Only by clarifying such basic issues can we further explore theoretical issues such as big data ownership, transaction, and capitalization and endow the digital economy with strong momentum for development.

The remainder of this paper is organized as follows. Section 2 gives a summary of the literature that has studied the transactions and taxation of data. In Section 3, a tax theory of data is given by illustrating the definition of data, the value of data and the taxability of data. In Section 4, the model of differentiated oligopolistic markets is introduced to analyze the tax system of data. Then, this paper studied the effects of ad valorem taxes and bot tax on the market. Subsequently, a comparison of the effects of the two types of taxes under quantity competition and price competition has been carried out. Finally, some concluding remarks and policy implications are provided in Section 5.

2. Literature Review

As an independent new factor of production, data are for the first time on par with traditional factors such as capital, labor, land, and technology. It is a major innovation in the theory of factor marketization (Brynjolfsson and Hitt 2006). As an important policy, the tax plays an important role in stimulating and regulating the development of data elements and is an important foundation, pillar and guarantee for the construction of the data element market (Aslam and Shah 2021).

Strictly speaking, data taxation is a new thing. It does not exist in the world at present, and its creation should be a breakthrough. At present, the international community is quite divided on

the issue of taxation of the digital economy. The Organization for Economic Cooperation and Development (OECD) has been quite proactive in its response and actions in the field of international taxation, especially in dealing with the impact of economic digitalization on international taxation rules (Lips 2019). Due to the complexity of the digital economy and the different demands of different countries in the chain of interests, the issue of taxation in the digital economy has not been able to form a consistent global response (Turina 2020).

With the rapid growth of the data scale and the rapid expansion of the digital economy, the scale of data assets will also continue to expand. Questions such as whether data assets can be taxed, which data assets can be taxed, and what are the core factors that prevent data assets from being taxed begin to emerge.

In recent years, scholars have actively explored the issues related to the taxation of data assets from the perspective of theory, systems and management. The value of the data is not reflected in the current tax system. The platform provides users with services or digital products, and users contribute data or content to the platform, which is a "barter" transaction to some extent (Kjærsgaard and Schmidt 2018). The platform provides data services to "exchange" users' data (Subramanian, Mitra et al. 2021), which will not be "deemed sales" under the current system and therefore not subject to taxation. At the same time, when the platform pays income tax, the value of the acquired data will not be included in the taxable income (Parsons 2021). Individuals provide data to the platform in exchange for the platform's services, but the services obtained are not currently classified as statutory income types and do not need to pay income tax (Devereux and Vella 2018). Because the tax system does not have a clear characterization of the data, the current data are still outside the taxation object (Gulkova, Karp et al. 2019). Even if the data obtain currency consideration during the transaction process, the current tax system does not have clear taxation regulations (Beaumier, Kalomeni et al. 2020). Effective tax management of data elements requires innovation; otherwise, the theoretically designed tax policy may be in an embarrassing situation that cannot be collected (Schneider 2018).

Data governance in the digital economy era needs to break through the boundaries of traditional tangible organizations and consider multiple levels from within and across industries, within and across regions, across the country, and across the globe (Shome 2021). The OECD believes that the development of the digital economy has made data more valuable, but no consideration has been paid during the production and use of data, so taxation is required (Förster, Greil et al. 2019). In the process of the globalization of the digital economy, there has been a dislocation between the digital economy and the traditional tax system (Asen and Bunn 2021). The transaction form of cross-border services has caused a mismatch between the place where profits are taxed and the place where value is created, which has become a new contradiction in the distribution of profits among countries (Becker and Englisch 2019).

From the perspective of the source of data assets, the evaluation government has natural administrative advantages in terms of source control, objective verification and functional analysis of data management, which can bring higher efficiency to public management and social services (Carriere-Swallow and Haksar 2019). However, existing research believes that this kind of tax initiated by a few countries lacks global system design and cooperation in terms of tax base, tax object, tax scope, etc. The highly targeted tax policy does not conform to economic principles, and double taxation increases corporate burdens and legal uncertainty (Lucas-Mas and Junquera-Varela 2021); in particular, a tax system that only targets companies outside the territory or in a specific country is a form of tax discrimination that

encourages protectionism (Sidik 2022). For developing countries, tax retaliation by developed countries will only reduce the quality of digital services and make it difficult to play their due role (Asen and Bunn 2021).

Simultaneously, the realization of the value of data assets faces technical problems and legal challenges (Katterbauer 2020). The tax department needs to take both technical and legal aspects to promote the transformation and reform of tax governance in the era of big data (Gulkova, Karp et al. 2019). From the current point of view, since the tax system of data elements is the basic system for establishing the circulation and use of data elements and the unique characteristics of data elements such as intangibility, reusability, and continuous value-added, the feasibility of the tax collection and management plan depends on determining the data assets (Shome 2021). The importance of concept and scope will be more prominent than other elements in the regulation of data taxation.

Missing from the literature is a clear and reasonable data property taxation system, which is the starting point of the logical analysis of data tax economics. In this paper, the data element and the tax system of data will be discussed. Data tax takes data elements as the taxation object and highlights the production element attributes of data resources. In the following section, the definition of data, the value of data and the taxability of data will be analyzed. Moreover, this paper also compares effects between ad valorem tax and bit tax on the market and the sensitivity of price or output to the two taxes between quantity and price competition.

3. Tax theory of data

3.1 Meaning of data in data tax

Data tax is a tax that takes data as the object of taxation. Data tax in a broad sense refers to all taxes levied on data, including various direct and indirect taxes levied on data; in a narrow sense, data tax refers to a special tax that takes data as the object of taxation.

Data tax is different from digital tax. Digital taxes have attracted much attention in the era of the digital economy. Digital in the digital economy has many connotations, which can be understood as data, that is, digital knowledge and information; it can also be understood as digital technology. Digital tax, which is often mentioned, is summarized into three different levels by some scholars: e-commerce tax, economic digital tax and digital service tax (Avi-Yonah and Fishbien 2020). Obviously, digital tax is a tax related to data or digital technology, which is different from the data tax with data as the object of taxation.

Data refer to the symbols that record and identify objective events. Numbers, symbols, characters, text, sound, image and video all belong to data. Under the background of the digital economy, all kinds of data often need to combine network technology and information to reflect their important value. Therefore, data often refer to the data resources in cyberspace. Data have three attributes: physical attributes, that is, the data exist in binary form in the storage medium; existence attributes, that is, data exist in the form of human perception; and information attributes, that is, data have specific meanings. Data are closely related to numbers and information. Data assets, digital assets and information assets are essentially data, focusing on the physical attributes, perceptual attributes and information attributes of data, respectively.

3.2 The value of data in the digital economy

In the era of the digital economy, data, as a key factor of production, have produced considerable economic value. However, according to the existing tax law, the data are not included in the

object of taxation, resulting in obvious tax burden differences between different factors of production, which also makes it difficult to reflect the income function of the traditional tax system.

The traditional tax system is compatible with the industrial economy. The value creation of the industrial economy mainly follows the value chain model. In this model, value is created in the linear process of production, circulation, distribution and consumption, and the in and out of material in the process link has a clear record of value. Value added tax and income tax are levied on the basis of accurately defining the income and expenditure of each link.

However, in the value creation process of the digital economy, the value network model and the value shop model are more common. In the value network model, platform enterprises rely on technology to connect various users and help end users conduct transactions. By joining the platform, users can directly enhance the network value of the platform, and users can also contribute value to the platform by forming specific content and providing relevant data. In the value store model, enterprises create value by providing customers with problem solutions. This process is inseparable from the support of the data. Only by relying on a large amount of data can enterprises find problems and put forward solutions for customers in time.

In the era of the digital economy, data have penetrated every industry and business functional field. The application of massive data will herald a new wave of productivity growth and consumer surplus. However, the value of the data is not reflected in the current tax system. The cyber platform provides users with services or digital products, and users contribute data or content to the platform. To some extent, it is a "barter" transaction. The platform provides data services to "exchange" users' data, which will not be "regarded as sales" under the current system, so there is no need to pay taxes. When the platform pays income tax, the value of the obtained data will not be included in taxable income. Individuals provide data "exchange" platform services to the platform, but the services obtained do not belong to the legal income type and do not need to pay income tax. Therefore, because the tax system does not have a clear qualitative definition of the data, these data are still separated from the object of taxation at present. Even if the data obtain monetary consideration in the transaction process, the current tax system has no clear tax provisions.

3.3 Taxability of data

It is necessary to investigate whether it is reasonable for the data to be separated from the object of taxation according to taxable theory. The theory of taxability points out the factors to be considered in determining the object of taxation, which is conducive to preventing arbitrary expansion or improper restriction of the object of taxation.

The income status of data plays a decisive role in the economic taxability of data. Generally, data are used as means of production, commodities, exchange media and other ways to generate value. However, the data exist in different states, whether the data generate income and the specific forms of income are different, and the taxability of the data is also different.

The first kind of data is retained data. When the data are kept and not used in production activities, it is impossible to generate income. Therefore, it is not economically taxable. When the retained data are put into production, it produces value, and the data are taxable, but its taxability needs to be judged according to the characteristics of income. If the value income created by data can be clearly measured, then these data are economically taxable. If the data are

put into use as a means of production and create value, but this part of the value is integrated into other goods or services, and the value attributed to the data is difficult to measure independently under the existing conditions, then this kind of data is not taxable.

The second kind of data is the data used as a nonmonetary asset exchange. There are two cases of nonmonetary exchange of data. The first kind is the exchange of data. The typical situation is that personalized data are exchanged with the data products and services of the platform. Due to the lack of monetary tools in the exchange process and the lack of objective standards to measure the value of various data, it is difficult to clearly measure the income of data. Therefore, the data in this case are not economically taxable. Another case is the exchange of data with other nonmonetary objects. If the other nonmonetary physical goods can be accurately priced under the current technical conditions, the nonmonetary physical goods obtained by data exchange belong to measurable income, and the data are economically taxable at this time.

The third kind of data is the data used as monetized transactions. Since money is used as the pricing scale of data in the transaction process, the income of data is directly reflected in the amount of money, which meets the requirements of income measurability. Therefore, data as monetized transactions are economically taxable.

4. Theories of data tax and their comparison

4.1 Calculating tax based on the amount of data

The first kind of data tax is calculated based on the amount of data transmitted through the network, also known as bit tax (Cordell 1997). Bit tax is an earlier theory of data information taxation, and its tax base is the amount of data transmitted through the network.

Under the condition of the digital economy, the increase in wealth value mainly comes from the interaction of data. Data create productivity in the network. The massive amount of data flowing on the network contains rich wealth. The value created by data penetrates into commodity production. Although it cannot be measured accurately, this kind of value does exist. Relatively speaking, in the era of the traditional economy, the value of each input and its contribution to output are easy to calculate. Therefore, value-added tax is an ideal tax; however, under the condition of the digital economy, it is difficult to calculate the value created by various elements. Therefore, bit tax can be used to replace the traditional tax, and the data transmission volume can replace the value increase as the tax base.

Collecting bit tax can increase the cost of transmitting garbage data and encourage users to use the network reasonably to reduce information pollution and alleviate network congestion. However, there are deficiencies in the principle of bit tax. First, the tax object has limitations and is not conducive to the development of the current digital economy. The bit tax only taxes data transmitted through the network, and those data that do not need to be transmitted through the network are excluded. In addition, taxing network data transmission behavior will inhibit the in-depth development of the digital economy. Third, bit tax ignores the value of the data itself, which easily leads to an unfair tax burden. The information content and data value of transmitting the same number of bits of data are likely to be different, so it is unfair to take the amount of data transmitted by the network as the tax base. In the context of the digital economy, data taxation is mainly due to the contribution of data in the process of value creation. Taxation must consider the value of data. In addition, according to the amount of data transmission, double taxation may exist when the same data are transmitted back and forth many times but

does not generate value.

4.2 Calculating tax based on the value of data

Another theory for levying data tax is to take data as the tax object and the property value of the data as the tax base, that is, to levy property tax on data (ref). The higher the valuation of the data, the more tax-affordable it is. Therefore, calculating the tax base based on the value of the data complies with the requirements for volume-based taxation.

Determining the value of the data becomes the key to taxation, which requires a valuation of the data. According to the principle of asset evaluation, three methods can generally be used to value assets. The first is based on the cost method, where the value of the data asset being valued can be determined at its current replacement cost. The second is based on the income method, which can determine the value of the data asset by estimating the present value of the expected future income of the data asset. The third is based on the market method, which can determine the value of data assets by comparing the similarities and differences between the data assets being assessed and similar assets sold recently and adjusting the market prices of similar assets.

4.3 Comparison of two taxes

In this discussion, we study the model used by Häckner (Häckner and Herzing 2016), which assumes that $n (\geq 2)$ firms run platforms for users and that the utility of each user is quadratic in the consumption of the digital goods itself in one platform and linear in the consumption of digital goods in other platforms. The utility function of the representative users is $U(q, I) = \alpha \sum_{j=1}^n q_j - \frac{1}{2} (\sum_{j=1}^n q_j^2 + 2r \sum_{j \neq i} q_i q_j) + I$, where $q = (q_1, q_2, \dots, q_n)$ is the quantity of digital goods provided by each firm, n is the number of symmetric firms active in the market that produce the digital goods, α is a demand-shifting parameter and is the same for each symmetric firm, and I is the budget constraint. In this discussion, we assume that the q -goods are symmetric, and the digital products are substitutable. The substitutability between digital products is measured by $r \in (0, 1)$. Representative consumers maximize their utility $U(q, I)$ subject to the budget constraint m . It can be written as $\sum_{i=1}^n p_i q_i + I \leq m$, where p_i is the price for q_i . The utility function is concave, and there exists a unique solution. Then, the first-order condition determining the optional consumption of good q produced by firm j is $\frac{\partial U}{\partial q_j} = \alpha - q_j - r \sum_{i \neq j} q_i - p_j = 0$. Therefore, firm j 's inverse demand function can be solved: $p_j(q_j, q_{-j}) = \alpha - q_j - r \sum_{i \neq j} q_i$, where $q_{-j} = (q_1, \dots, q_{j-1}, q_{j+1}, \dots, q_n)$.

Let τ ($0 < \tau < 1$) and t ($t > 0$) denote the levels of ad valorem taxes and bit taxes, c denotes the constant marginal cost of production, which is the same across these firms, and k denotes the fixed cost. For firm j , the profit function is $\pi_j(p_j, q_j) = [(1 - \tau)p_j - c - t]q_j - k$.

In the following section, we conduct a number of experiments to analyze how various market characteristics affect the market structure. Now, we further illustrate the model under different modes of competition (quantity competition and price competition).

Quantity competition

If all enterprises compete according to the quantity of products they produce, this mode is called Cournot competition. We derive the price p^{C*} , the demand q^{C*} and the profit π^{C*} under quantity competition. In this equilibrium, p_j is determined by q_j in the profit function. Therefore, for firm

j , its profits are $\pi_j(p_j(q_j), q_j) = [(1 - \tau)(\alpha - q_j - r \sum_{i \neq j} q_i) - c - t]q_j - k$. Firms decide quantities to maximize their profits $\pi_j(n)$, and this situation can be described as $\frac{d\pi_j(p_j(q_j), q_j)}{dq_j} = [(1 - \tau)(\alpha - 2q_j - r \sum_{i \neq j} q_i) - c - t]$. The first-order conditions yield $q_j(q_{-j}) = \frac{1}{2}(\alpha - \frac{c+t}{1-\tau} - r \sum_{i \neq j} q_i)$. By summing over of the all firms and using the relation that $\sum_{i=1}^n q_i = q_j + \sum_{i \neq j} q_i$, it yields the following result: $q^{C*} = \frac{\alpha - \frac{c+t}{1-\tau}}{2+r(n-1)}$, $p^{C*} = \alpha - [1 + r(n-1)] \frac{\alpha - \frac{c+t}{1-\tau}}{2+r(n-1)}$, $\pi^{C*} = \frac{(1-\tau)(\alpha - \frac{c+t}{1-\tau})^2}{[2+r(n-1)]^2} - k$.

Effects of taxation under quantity competition on the market

We are first concerned with the market situation under quantity competition. To solve this problem, we need to understand the output decisions of competing but not coordinated manufacturers in the state of Cournot equilibrium.

Let $n_0^{C*} (n_0^{C*} \geq 2)$ denote the maximal number of firms that remain in the market such that the profit of each firm is nonnegative. Such n_0^{C*} exists because $\pi^{C*} = \pi^{C*}(n, \tau, t)$ is monotonically decreasing with respect to n . Here, for simplicity, we no longer regard n_0^{C*} as an integer but define n_0^{C*} as a number satisfying $\pi^{C*}(n_0^{C*}, \tau, t) = 0$.

Furthermore, we discuss the total output Q^{C*} of the industry when the number of firms is maximal. Note that we only consider the case $r > 0$, which means that the digital products of firms have substitutability.

Proposition 1. *Under quantity competition, when all firms reach Cournot equilibrium, if*

$\pi^{C*}(2, \tau, t) = \frac{(1-\tau)(\alpha - \frac{c+t}{1-\tau})^2}{(2+r)^2} - k \geq 0$, *then the number of retained companies in the market n_0^{C*} and corresponding total output Q^{C*} decrease when the tax rate τ or t increases.*

Proof. The complete proof of Proposition 1 is provided in Appendix B. \square

Tax will increase the burdens of enterprises, reduce profits, and consequently restrain the development of the corresponding industry. Whether it is a tax on an enterprise or a tax on consumers, it will inhibit market activity and reduce the volume of transactions. Therefore, the market saturation of the industry will be reduced accordingly. If an industry over expands, a tax increase can be adopted to restrain its growth, while if an industry is not sufficiently developed, tax relief or subsidies can be adopted to stimulate its growth. Therefore, taxation is an important means of promoting the adjustment, optimization, and upgrading of industrial structures.

Comparison of the two taxes under quantity competition

In this subsection, we compare the effects of the different taxes on output q^{C*} or price p^{C*} under quantity competition; that is, these indexes will be discussed under Cournot equilibrium. Consequently, in this section, we assume the taxes τ, t to be changing variables and α, c, r, n to be constants.

Note that the partial derivatives $\begin{cases} \frac{\partial q^{C*}}{\partial \tau} = \frac{1}{2+r(n-1)} \left(-\frac{c+t}{(1-\tau)^2} \right) \\ \frac{\partial q^{C*}}{\partial t} = \frac{1}{2+r(n-1)} \left(-\frac{1}{1-\tau} \right) \end{cases}$ can be rewritten as

$$\begin{cases} \frac{\partial p^{C^*}}{\partial \tau} = \frac{1+r(n-1)}{2+r(n-1)} \frac{c+t}{(1-\tau)^2} \\ \frac{\partial p^{C^*}}{\partial t} = \frac{1+r(n-1)}{2+r(n-1)} \frac{1}{1-\tau} \end{cases} . \text{ These imply that when one of the taxes increases, the other is fixed.}$$

Consequently, the output will decrease, and the price will increase.

However, to compare the effects of two taxes, we should increase one tax and decrease the other such that the total tax revenue is fixed, i.e., $(\tau p^{C^*} + t)q^{C^*}n = C_0$ for some (τ, t) . Here, C_0 is a constant. Since q^{C^*} and p^{C^*} are functions of (τ, t) , the above equations actually determine a relation of τ and t . Therefore, with the above fixed total tax revenue, both q^{C^*} and p^{C^*} are functions of one variable τ . Now, we check the derivatives $\frac{dq^{C^*}}{d\tau}$ and $\frac{dp^{C^*}}{d\tau}$. If the sign of $\frac{dq^{C^*}}{d\tau}$ (or $\frac{dp^{C^*}}{d\tau}$) is the same as the sign of $\frac{\partial q^{C^*}}{\partial \tau}$ (or $\frac{\partial p^{C^*}}{\partial \tau}$), the output (price) is more sensitive to τ than to t . If the sign of $\frac{dq^{C^*}}{d\tau}$ (or $\frac{dp^{C^*}}{d\tau}$) is the opposite of the sign of $\frac{\partial q^{C^*}}{\partial \tau}$ (or $\frac{\partial p^{C^*}}{\partial \tau}$), then the output (price) is more sensitive to t than to τ .

Note that it is not guaranteed that the above equation determines a unique function $t = t(\tau)$ for arbitrary constant C_0 . Therefore, we should focus on a small neighborhood of a given (τ_0, t_0) , i.e., consider (τ, t) in a small neighborhood of (τ_0, t_0) such that $[\tau p^{C^*}(\tau, t) + t]q^{C^*}(\tau, t)n = [\tau_0 p^{C^*}(\tau_0, t_0) + t_0]q^{C^*}(\tau_0, t_0)n$ and use the implicit function theorem.

Proposition 2. *Under quantity competition, when all firms reach Cournot equilibrium, given any (τ_0, t_0) satisfying $-2[2 + r(n - 1) - \tau_0] \frac{\alpha - c + t_0}{1 - \tau_0} + \alpha - c < 0$, there exists a differentiable function $t = t(\tau)$ such that $(\tau, t(\tau))$ is in a small neighborhood of (τ_0, t_0) , $t_0 = t(\tau_0)$, and the condition of fixed total tax revenue [total tax I] is satisfied. For $t = t(\tau)$, if τ increases, then t decreases.*

Furthermore, the output is more sensitive to t than to τ , while the output is more sensitive to τ than to t .

Proof. The complete proof of Proposition 2 is provided in Appendix C. \square

This proposition leads to the following conclusion. With a fixed total tax revenue and some conditions on two types of taxes, an increase in one tax will result in a decrease in the other. Moreover, under the above assumption, it is found that the output is more sensitive to bit taxes than to ad valorem taxes and that the price is more sensitive to ad valorem taxes than bit taxes.

Price competition

We are now concerned with the market situation under price competition. If all enterprises compete according to the price of products they produce, this mode is called Bertrand competition. To solve this problem, we need to understand the output decisions of competing but not coordinated manufacturers in the state of Bertrand equilibrium.

In this part, we derive the price p^{C^*} , the demand q^{C^*} and the profit π^{C^*} under Bertrand competition. Summing over all of the firms in their inverse demand function yields $\sum_{j=1}^n p_j = n\alpha - \sum_{j=1}^n q_j - r(n-1) \sum_{j=1}^n q_j$, Firm j 's demand function is. Under Bertrand competition, firms decide prices to maximize profits: $\pi_j(p_j, q_j(p_j))$, $\frac{d\pi_j(p_j, q_j(p_j))}{dp_j} = (1 - \tau)q_j +$

$$[(1 - \tau)p_j - c - t] \frac{dq_j}{dp_j} = 0, p_j(p_{-j}) = \frac{(1-r)\alpha + r \sum_{i \neq j} p_i}{2[1+r(n-2)]} + \frac{c+t}{2(1-\tau)}$$

Summing over all of the firms yields

$$p^{B*} = \frac{1-r}{2+r(n-3)} \alpha + \frac{1+r(n-2)}{2+r(n-3)} \frac{c+t}{1-\tau}, \quad q^{B*} = \frac{1+r(n-2)}{[1+r(n-1)][2+r(n-3)]} \left(\alpha - \frac{c+t}{1-\tau} \right),$$

$$\pi^{B*} = \frac{(1-r)[1+r(n-2)]}{[1+r(n-1)][2+r(n-3)]^2} (1 - \tau) \left(\alpha - \frac{c+t}{1-\tau} \right)^2 - k.$$

The complete calculations of q_j , p^{B*} and q^{B*} are provided in Appendix A.

Effects of taxation under price competition on the market

We are now concerned with the market situation under price competition. To solve this problem, we need to understand the output decisions of competing but not coordinated manufacturers in the state of Bertrand equilibrium. Market situation can be analyzed by calculating the maximal number of firms $n_0^{B*} (\geq 2)$ such that the profit of each firm is nonnegative. Here, for simplicity, we no longer treat n_0^{B*} as an integer but define n_0^{B*} as the number satisfying $\pi^{B*}(n_0^{B*}, \tau, t) = 0$. Here, we only consider the case $r > 0$.

Proposition 3. *Under quantity competition, when all firms reach Bertrand equilibrium, if $\pi^{B*}(2, t, \tau) = \frac{(1-r)}{(1+r)(2-r)^2} (1 - \tau) \left(\alpha - \frac{c+t}{1-\tau} \right)^2 - k \geq 0$, then the number of retained companies in the market n_0^{B*} exists and decreases when tax rate τ or t increases.*

Proof. Let $y := 1 + r(n_0^{B*} - 2)$; then, $\pi^{B*}(n_0^{B*}, \tau, t) = 0$ implies $\frac{(1-r)y}{(y+r)(y+1-r)^2} (1 - \tau) \left(\alpha - \frac{c+t}{1-\tau} \right)^2 = k$, which means $(y + r)(y + 1 - r)^2 = \frac{1-r}{k} (1 - \tau) \left(\alpha - \frac{c+t}{1-\tau} \right)^2 y$.

For simplicity, denote $\theta := \frac{1-r}{k} (1 - \tau) \left(\alpha - \frac{c+t}{1-\tau} \right)^2$; thus, the above equation reduces to $(y + r)(y + 1 - r)^2 = \theta y$. Since $\pi^{B*}(2, \tau, t) > 0$, $(y + r)(y + 1 - r)^2 \leq \theta y$ when $y = 1$. Denote that $f(y) := (y + r)(y + 1 - r)^2$ and $g(y) := \theta y$. Then, $f(0) > 0 = g(0)$, $f(1) - g(1) < 0$, $f(y) - g(y) \rightarrow +\infty$ as $y \rightarrow +\infty$, and $f(y) - g(y) \rightarrow -\infty$ as $y \rightarrow -\infty$. The cubic $(y + r)(y + 1 - r)^2 = \theta y$ has three roots lying in $(-\infty, 0)$, $(0, 1)$ and $(1, +\infty)$ or two roots lying in $(-\infty, 0)$ and at $y = 1$. Consequently, the graph of the functions on the left-hand (blue line) and right-hand (red line) sides of the above equation should be as in Figure 1.

Figure 1 indicates that $n_0^{B*} \geq 2$ must exist. Obviously, when t or τ increases, the slope θ decreases; thus, the saturation of the enterprises in industry n_0^{B*} decreases. Therefore, tax revenue restrains market saturation. □

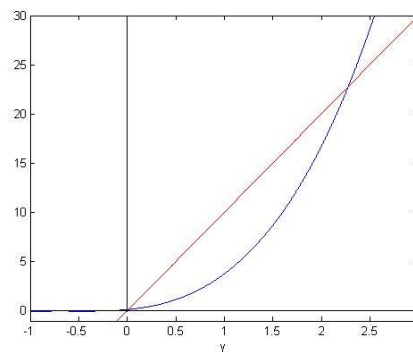


Figure 1: The graph of $f(y) := (y + r)(y + 1 - r)^2$ and $g(y) := \theta y$.

In the above proposition, we derive the same conclusion as that under Cournot competition (Proposition 1); i.e., under Bertrand competition, both ad valorem and bit taxes restrain market saturation.

We now compare a regime with the two taxes under Bertrand equilibrium.

Comparison of the two taxes under Bertrand equilibrium

We assume the taxes τ, t to be changing variables and α, c, r, n to be constants.

Note that the partial derivatives $\begin{cases} \frac{\partial q^{B^*}}{\partial \tau} = \frac{1+r(n-2)}{[1+r(n-1)][2+r(n-3)]} \left(-\frac{c+t}{(1-\tau)^2} \right), \\ \frac{\partial q^{B^*}}{\partial t} = \frac{1+r(n-2)}{[1+r(n-1)][2+r(n-3)]} \left(-\frac{1}{1-\tau} \right), \end{cases}$ which are

$\begin{cases} \frac{\partial p^{B^*}}{\partial \tau} = \frac{1+r(n-2)}{2+r(n-3)} \frac{c+t}{(1-\tau)^2}, \\ \frac{\partial p^{B^*}}{\partial t} = \frac{1+r(n-2)}{2+r(n-3)} \frac{1}{1-\tau}. \end{cases}$ These imply that when one of the taxes increases and the other is fixed, the output will decrease, and the price will increase.

Similar to the arguments in the comparison under quantity competition (Proposition 2), we should focus on a small neighborhood of a given (τ_0, t_0) , i.e., consider (τ, t) in a small neighborhood of (τ_0, t_0) such that $[\tau p^{B^*}(\tau, t) + t]q^{B^*}(\tau, t)n = [\tau_0 p^{B^*}(\tau_0, t_0) + t_0]q^{B^*}(\tau_0, t_0)n$. Under this condition, both q^{B^*} and p^{B^*} are functions of one variable τ . When one tax increases and the other decreases such that the total tax revenue is fixed, if the sign of $\frac{dq^{B^*}}{d\tau}$ (or $\frac{dp^{B^*}}{d\tau}$) is the same as the sign of $\frac{\partial q^{B^*}}{\partial \tau}$ (or $\frac{\partial p^{B^*}}{\partial \tau}$), then the output (price) is more sensitive to τ than to t . If the sign of $\frac{dq^{B^*}}{d\tau}$ (or $\frac{dp^{B^*}}{d\tau}$) is the opposite to the sign of $\frac{\partial q^{B^*}}{\partial \tau}$ (or $\frac{\partial p^{B^*}}{\partial \tau}$), then the output (price) is more sensitive to t than to τ .

Proposition 4. *Under quantity competition, when all firms reach Bertrand competition, given any (τ_0, t_0) satisfying $-2[1+r(n-1) + \frac{[1+r(n-1)](1-r)}{1+r(n-2)}(1-\tau_0)]q^{B^*}(\tau_0, t_0) + \alpha - c < 0$, there exists a differentiable function $t = t(\tau)$ such that $(\tau, t(\tau))$ is in a small neighborhood of (τ_0, t_0) , $t_0 = t(\tau_0)$, and the condition of fixed total tax revenue $[\tau p^{B^*}(\tau, t) + t]q^{B^*}(\tau, t)n = [\tau_0 p^{B^*}(\tau_0, t_0) + t_0]q^{B^*}(\tau_0, t_0)n$ is satisfied. For $t = t(\tau)$, if τ increases, then t decreases.*

Furthermore, the output is more sensitive to t than to τ , while the output is more sensitive to τ than to t .

Proof. The complete proof of Proposition 4 is provided in Appendix D. \square

In the above proposition, we derive the same conclusion as that under Cournot competition (Proposition 2); i.e., under Bertrand competition, with a fixed total tax revenue and a condition of two types of taxes, an increase in one tax will result in a decrease in the other tax. Under the above assumption, it is found that the output is more sensitive to bit taxes than to ad valorem taxes, while the price is more sensitive to ad valorem taxes than to bit taxes.

Comparisons between Cournot equilibrium and Bertrand equilibrium

Comparisons of the outputs, prices, and profit levels between Cournot equilibrium and Bertrand equilibrium were studied. It was proven that when goods are substitutes under Cournot

competition, the outputs are lower, and the prices and the profits are higher than those under Bertrand competition. We compare the number of retained companies and the effects of the two taxes on the price, output, and profit level under the two types of competition between the two competitors. The results are as follows:

Proposition 5. For given τ , t , n , if $r > 0$, then $n_0^{C*} > n_0^{B*}$.

Proof. The complete proof of Proposition 5 is provided in Appendix E. \square

When the rates of ad valorem and bit taxes are provided and both of the competitions reach their saturations, the number of enterprises in price competition is smaller than that in quantity competition; that is, the market saturation under Bertrand competition is less than that under Cournot competition.

As proven in Proposition 4, the profit in price competition is greater than that in production competition. A company obviously has a greater chance of survival when it has greater profits.

Proposition 6. If $r > 0$, then $0 > \frac{\partial q^{C*}}{\partial t} > \frac{\partial q^{B*}}{\partial t}$, $0 > \frac{\partial q^{C*}}{\partial \tau} > \frac{\partial q^{B*}}{\partial \tau}$, $0 < \frac{\partial p^{C*}}{\partial t} < \frac{\partial p^{B*}}{\partial t}$, $0 < \frac{\partial p^{C*}}{\partial \tau} < \frac{\partial p^{B*}}{\partial \tau}$.

Proof. These inequalities can be directly obtained by the formulas $\frac{\partial q^{C*}}{\partial t}$, $\frac{\partial q^{B*}}{\partial t}$, $\frac{\partial p^{C*}}{\partial t}$, $\frac{\partial p^{B*}}{\partial t}$ and (E.1). \square

Remarks According to the above comparison result, production and price are less sensitive to both taxes under Cournot competition than under Bertrand competition.

Cournot competition is monopolistic, and the marginal cost must be charged. However, as the number of companies in Cournot competition approaches infinity, the whole economy approaches perfect competition. In other words, the Cournot model is truly looking for profit.

In the Bertrand model, we can acquire exactly the same results from the derivation and can even change the proceeds to a certain value and limit the lowest price for the cost price. In short, Bertrand is a fancy name for perfect competition, the price of which is the marginal cost. In other words, the Bertrand model is essentially not the pursuit of profit but the pursuit of the most customers without losing money.

The reason is the unrealistic assumption of the Bertrand model itself that consumers in the world are well informed, there are no errors and no delays of information, there are no technology or resource barriers between companies, and a new product can be a perfect knock-off and will not run out of stock.

The results of Propositions 5 and 6 show that, compared with Bertrand equilibrium, under Cournot equilibrium, market saturation is greater, and the effects of the two taxes on output or price are all smaller.

From the perspective of government, maximizing social welfare is one of the most important aims when framing economic policies. The welfare dominance of ad valorem taxes under Cournot behavior in a single-product oligopoly for heterogeneous-cost firms is confirmed.

5. Concluding remarks

This analysis has discussed the taxation of big data and focused on the explicit comparison of key measures under ad valorem and bit taxation. It has been shown that under both quantity competition (Cournot equilibrium) and (Bertrand equilibrium), bit taxes dominate ad valorem

taxes in terms of production, while ad valorem taxes dominate bit taxes on price. Moreover, both output and price are less sensitive to bit taxes and ad valorem taxes under quantity competition than under price competition.

Unfortunately, this article has two major limitations. First, we were unable to obtain enough data for empirical research. Second, we assume that the total tax amount is fixed. To simplify the model, we ignore complex cases, which will be discussed further in our future research.

The content of the collection of digital tax includes the transformation and improvement of tax types, the scope of collection, the method of collection, and the overall adjustment of taxes and fees. This article only focuses on the changes in collection methods in the resource tax reform.

Another theory of levying data tax is to collect tax data based on the income obtained by using data¹⁶. The benefits of data include the benefits of retention and circulation of data. Additionally, monetized income and no monetized income are included. For example, individuals provide data to enterprises on various platforms and obtain relevant services from enterprises. The service income created by these personal data is also a potential tax base or tax source.

A data tax levied on the basis of data revenue is in line with the taxability principle of taxation. Data can generate revenue; thus, it has tax affordability. To levy data tax, the data that can generate income should be the tax object, and the income should be the tax base. The data tax, which calculates the tax base based on the value of data, reflects the principle of taxation by volume and is conducive to the realization of tax fairness. The higher the value added from the data, the higher the tax affordability. Therefore, calculating the tax base based on the value of the data complies with the requirements for volume-based taxation.

A question that naturally arises is how the government can come up with a better mix of tax administration, as it wishes to maximize consumer welfare (consumer surplus plus profit) given some specific tax requirements. In the future, we can continue to analyze the impact of tax changes by considering the above factors.

Acknowledge

This work was partially supported by the Special Research Project on Ideological and Political Theory Courses in Colleges and Universities funded by the National Social Science Foundation of PRC (21VSY077) and the Guangdong Planning Office of Philosophy and Social Science (GD20YYJ01).

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