Technique for Solving the Problem of Interpolation of Functions using the Mathematical Package MathCAD

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Annotation

Many of those who deal with scientific and engineering calculations often have to work with sets of values obtained empirically or by random sampling. As a rule, on the basis of these sets, it is required to construct a function on which other obtained values could fall with high accuracy. Such a task is called approximation. Interpolation is a type of approximation in which the curve of the constructed function passes exactly through the available data points.

This article discusses the problem of interpolation of functions in tabular form using the MathCAD mathematical package. The information presented in the article can be used when performing laboratory classes in such academic disciplines as "Numerical methods", "Computer modeling".

Keywords: function, interpolation, table, algebraic polynomial, MathCAD, vectors.

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When solving many technical problems, it is required to obtain a functional dependence of one quantity on another. In rare cases, it is possible to obtain such dependence analytically. Experimental studies of a technical object are carried out much more often. In this case, a certain value of the control parameter (the value of the argument x) is set and the corresponding value of the output (investigated) parameter (the value of the function y) is measured. In such cases, the experimental results are the values of the value y_i for some values of the argument x_i, where $i = 0, 1, 2, ..., n$. We can say that the function is given tabularly. If we display these values on the $x - y$ coordinate plane, we get a system of points (i.e. we can say that the function is set point-wise). However, for subsequent studies, it is necessary to obtain a functional dependence $y(x)$, by which it would be possible to determine the value of y with some accuracy for any value of the argument x. Obtaining such dependence in computational mathematics is called solving the problem of approximation (approximation) of a tabular (point-by-point) given function.

One of the most common ways to approximate functions is interpolation. It is used when the basic information about the approximate function is given in the form of a table of its values, and the tabular values have minor errors (the accuracy of the experiment is quite high). As a result of solving the interpolation problem, the line corresponding to the interpolating function will necessarily pass through all the points of the source data. In this case, the points are the interpolation nodes. Of all the possible interpolation options, in this article we will consider only linear and polynomial interpolation.

Let the function $y = f(x)$ be defined on a given segment $[a, b]$. The values of the function $y_i = f(x_i)$ are known at individual points x_i $(i = 0, 1, ..., n)$ of this segment. Calculating the values of this function at other points of the segment $[a, b]$ is either very laborious or impossible at all. In such conditions, one usually tries to obtain an approximate dependence instead of the function $f(x)$, which could be used to calculate approximate values of the function at other points of the segment $[a, b]$. The approximation of the function $f(x)$ on the segment $[a, b]$ means some other function $\varphi(x)$ defined on this segment $[a, b]$, whose values are sufficiently close to the corresponding values of the function $f(x)$ ($f(x) \approx \varphi(x)$). The function $\varphi(x)$ satisfying the interpolation conditions is called the interpolation function, and the points $x_0, x_1, x_2, ..., x_n$ are the interpolation nodes [1].

Most often, algebraic polynomials are chosen as interpolation functions, because their values are calculated the easiest. Thus, the following problem is solved - an algebraic polynomial of the n – degree is defined in the following form

$$P_n(x_i) = f(x_i), \quad i = 0, 1, ..., n \tag{1}$$

Which satisfies the interpolation conditions [2]:

$$P_n(x) = a_0 + a_1 x + a_2 x^2 + ... + a_n x^n \tag{2}$$

An algebraic polynomial satisfying these conditions is called an interpolation polynomial. Now let's consider the solution of the problem of interpolation of a function presented in tabular form in the MathCAD system [3].

Table 1

x	0,1	1	2	3	4	5	6	7	8	9
y	20,2	7,84	5,94	4,01	3,77	4,23	3,38	3,03	3,04	3,15

It is required to construct linear and polynomial (ninth degree) interpolating functions. In addition, it is necessary to calculate the values of the function using interpolating and approximating formulas for some values of the argument that do not coincide with its values from Table 1. These are the argument values: 1,3; 2,6; 4,4; 5,9; 7,1; 8,75.

To solve the problem, the initial data contained in Table 1 will be displayed in MATHCAD in the form of a table of x and y values, separate vectors of x and y values and a corresponding set of points on the $x - y$ plane. To do this, we will set a table named "Nuqtalar" as a 10×2 matrix and fill it with data from Table 1.
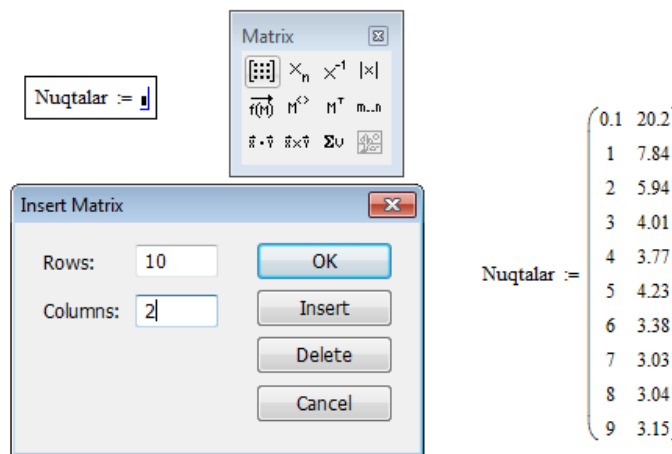

Fig.1. Assigning array sizes and filled array of values

Next, vectors of x and y values are allocated separately from the "Nuqtalar" array using the procedure for selecting columns from the matrix.
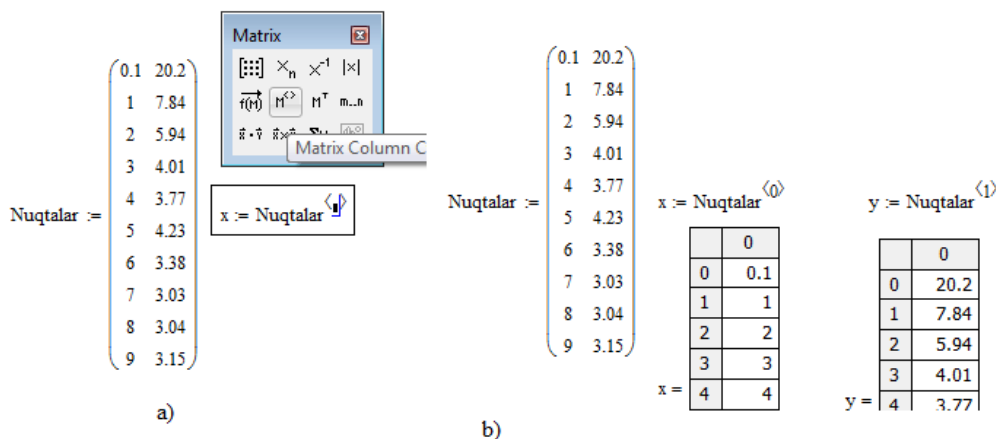

Fig.2. Selection of values of vectors x and y from the table (a) and checking the correctness of this action (b)

Next, a diagram is constructed in which the values from the vector x are plotted along the argument axis, and the values of the function from the vector y are plotted along the vertical axis.
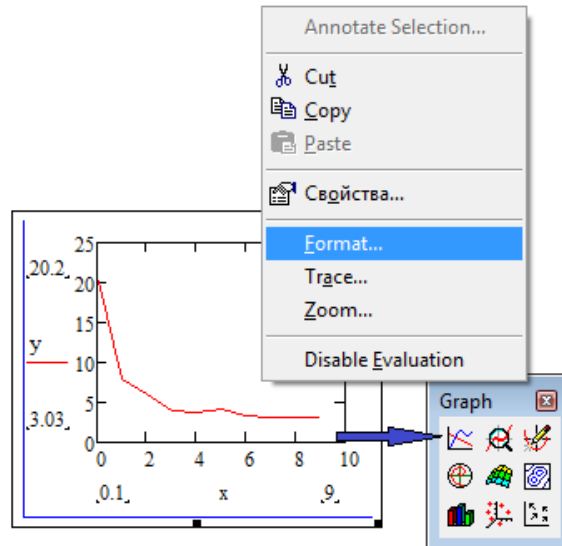

Fig.3. Plotting a diagram based on the values of the x and y vectors

To convert the diagram to a system of points (this is how the initial information is set), hover the cursor over any part of the line and click on the "Format" line on the panel that appears. In the "Formatting currently selected X-Y plot" panel that appears, select the "Traces" tab and assign the parameters "Symbol", "Symbol weight" and the absence of a line in the "Line" column for curve 1 on it. Then only the point system will be visible on the diagram.
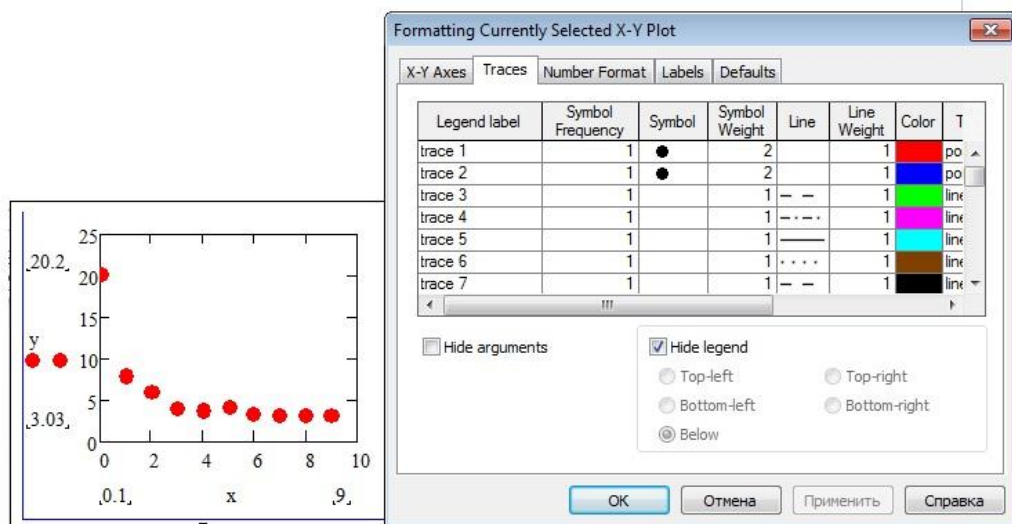

Fig.4. Converting a diagram to a "system of x and y points"

The following steps require drawing several more lines on the diagram, interpolating and approximating the system of points. To do this, a discrete variable xx is set, varying from the first value x_0 to the last value x_9 in increments of 0,1. To do this, use the button to select an

element of the vector x by index and the button to set the range of values of the variable.
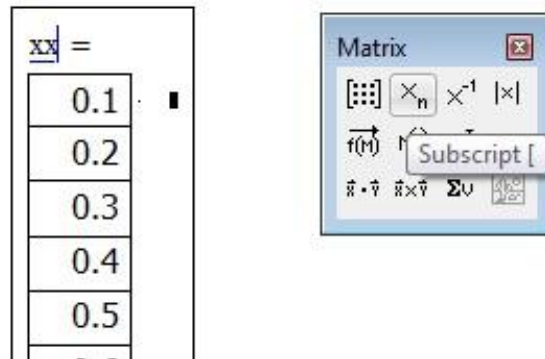
$$xx := x_0, x_0 + 0.1 .. x_9$$

Fig.5. Assignment of a discrete variable xx in increments of 0, 1

To implement linear interpolation in MATHCAD, there is a procedure "linterp (x,y,xx)", the arguments of which are a system of points (x_i, y_i) and discrete variables xx [3]. The results of this interpolation are shown in Fig. 6. It can be seen that the starting points are connected by sections of straight lines.

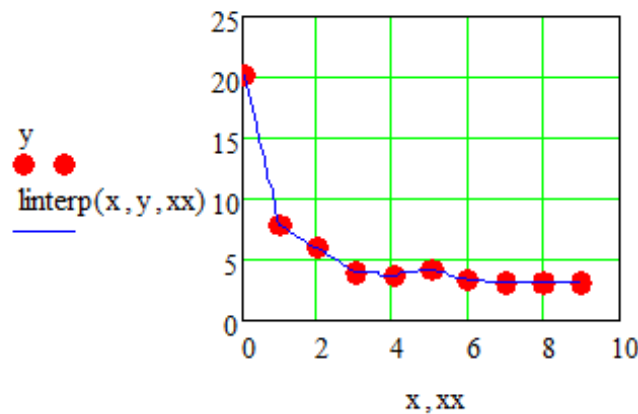$$xx := x_0, x_0 + 0.1 .. x_9$$

Fig.6. Linear interpolation of point source data

To implement the representation of point data by a polynomial function in MATHCAD, there is a procedure "regress(x,y,n)"[1]. Taking into account that 10 points are given, and then the interpolating should be of the 9th degree. The results of the "regression" procedure are contained in the vector z, and starting from the 3rd component (we remind you that indexing of array components in MATHCAD starts from 0), this vector contains the values of the coefficients of the polynomial (1).
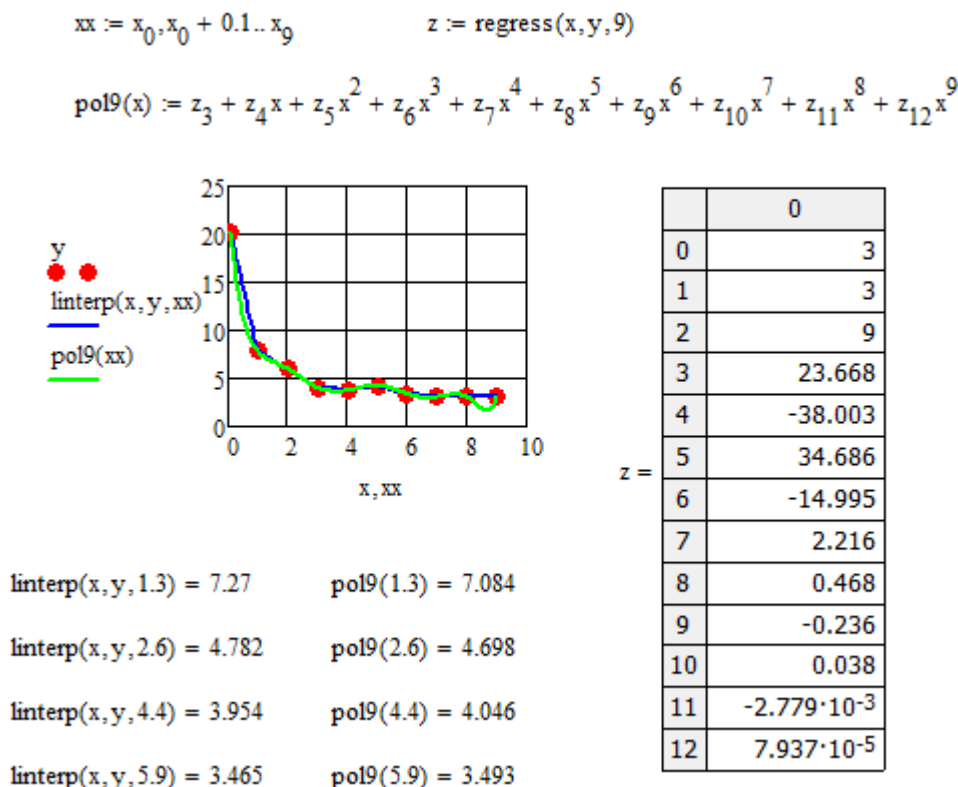
$$xx := x_0, x_0 + 0.1 .. x_9 \qquad z := \text{regress}(x, y, 9)$$

$$\text{pol9}(x) := z_3 + z_4 x + z_5 x^2 + z_6 x^3 + z_7 x^4 + z_8 x^5 + z_9 x^6 + z_{10} x^7 + z_{11} x^8 + z_{12} x^9$$

	0
0	3
1	3
2	9
3	23.668
4	-38.003
5	34.686
6	-14.995
7	2.216
8	0.468
9	-0.236
10	0.038
11	$-2.779 \cdot 10^{-3}$
12	$7.937 \cdot 10^{-5}$

$z =$

$\text{linterp}(x, y, 1.3) = 7.27 \qquad \text{pol9}(1.3) = 7.084$

$\text{linterp}(x, y, 2.6) = 4.782 \qquad \text{pol9}(2.6) = 4.698$

$\text{linterp}(x, y, 4.4) = 3.954 \qquad \text{pol9}(4.4) = 4.046$

$\text{linterp}(x, y, 5.9) = 3.465 \qquad \text{pol9}(5.9) = 3.493$

Fig.7. Linear and polynomial (9th degree) interpolation of point source data

As a result, we obtained a polynomial whose graph passes through all the interpolation nodes. The function values are also given for the given argument values that do not match the original table values. The calculations were performed using the linear interpolating function "linterp" and the interpolating formula for the 9th degree polynomial. It can be seen that the differences are beginning to manifest themselves in the last sections. The reason for this is the poor conditionality of the solution of the polynomial interpolation problem. For this reason, in other programs, polynomial interpolation is allowed for smaller values of the degree of the polynomial.

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